GRE

ARITHMETIC & ALGEBRA

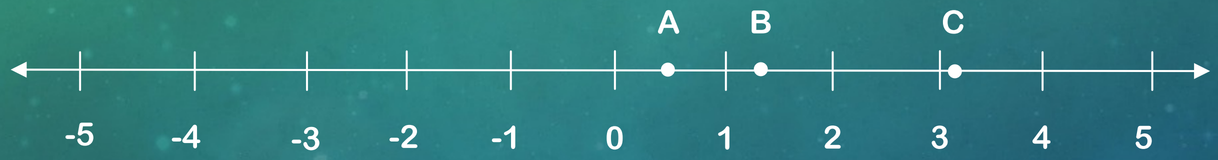
SHAYESTEH.IR

2021

GRE COURSE SECTIONS

1. Introduction
2. Quantitative Reasoning
    1. Quantitative Questions Formats
    2. Arithmetic & Algebra
    3. Geometry
    4. Data Interpretation
3. Verbal Reasoning
    1. Verbal Questions Formats (Sentence Equivalence & Text Completion)
    2. Reading Comprehension
4. Analytical Writing
    1. General Writing
    2. Argument Writing
    3. Issue Writing
5. General Writing
    1. Writing Ideas
    2. Cohesion & Coherence
    3. Writing Sentence Variety
    4. Grammar
    5. Punctuation
    6. The magic of 3

NUMBER PROPERTIES
THE NUMBER LINE



$$A = \frac{1}{2}$$

$$B = \sqrt{2}$$

$$C = 3.1$$

NUMBER PROPERTIES
THE REAL NUMBERS

Real Numbers = $\mathbb{R} = {}^{1}/_{2}\ , 3\ , 2.3\ , -6\ , -\sqrt{5}\ , -9.43\ , \ldots$

Integers = $\mathbb{Z} = \ldots , -2 , -1 , 0 , 1 , 2 , \ldots\ (0\ in\ not\ negative\ nor\ positive)$

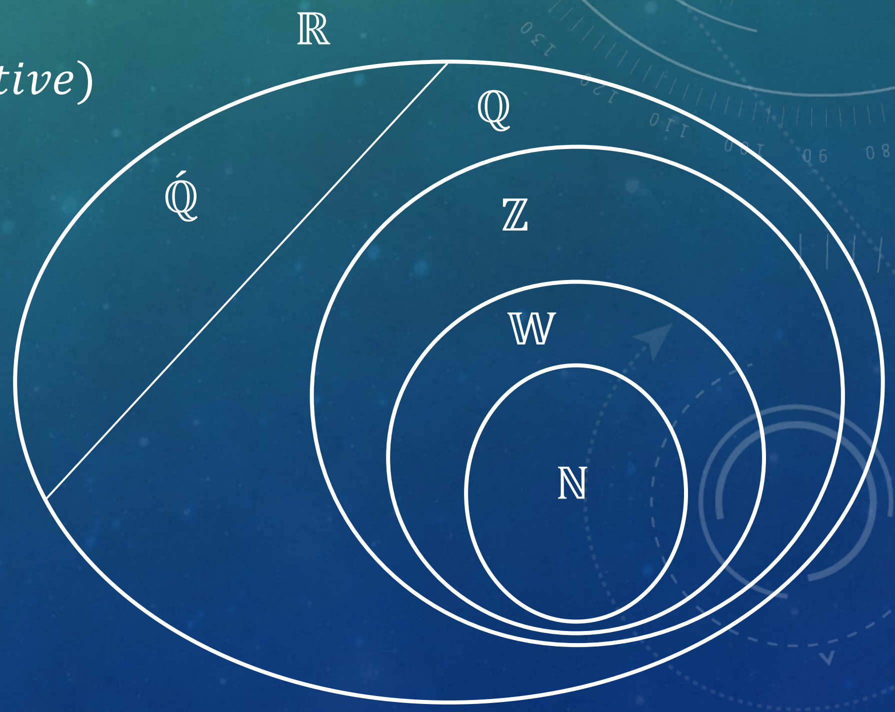
x, y, and z are positive integers means x, y, z >= 1

Natural Numbers = Counting Numbers = $\mathbb{N} = 1 , 2 , 3 , 4 , 5 , \ldots$

Whole Numbers = $\mathbb{W} = 0 , 1 , 2 , 3 , 4 , 5 , \ldots$

Rational Numbers = $\mathbb{Q} = \frac{2}{5} , \frac{6}{10} , \ldots$ $\qquad \frac{1}{3} = 0.333$

Irrational Numbers = $\acute{\mathbb{Q}} = \pm\sqrt{2}\ ,\ \pm\sqrt{3} , \pi , \ldots$



NUMBER PROPERTIES
ROUNDING NUMBERS

Example:

Consider the number 2,643.718

A. Round the number to the nearest tenth. = 2,643.7

B. Round the number to the nearest hundredth. = 2,643.72

C. Round the number to the nearest thousand. = 3,000

D. Round the number to the nearest unit. = 2,644

E. Round the number to the nearest hundred. = 2,600

NUMBER PROPERTIES
EXPANDED NOTATION

Example:

$$357 = 300 + 50 + 7 = 3\times10^2 + 5\times10^1 + 7\times10^0$$

$$568 = 500 + 60 + 8 = 5\times10^2 + 6\times10^1 + 8\times10^0$$

$$21{,}000 = 20{,}000 + 1{,}000 = 2\times10^4 + 1\times10^3$$

$$0.352 = 0.3 + 0.05 + 0.002 = 3\times10^{-1} + 5\times10^{-2} + 2\times10^{-3}$$

NUMBER PROPERTIES
PROPERTIES OF ZERO

- For any real number a, $a \times 0 = 0$.
- For any real number a, $a + 0 = a$.
- For any real number a, $a - 0 = a$ and $0 - a = -a$.
- For any nonzero real number a, $0 \div a = 0$.
- For real numbers a and b, if $a \times b = 0$, then $a = 0$ or $b = 0$.

NUMBER PROPERTIES
PROPERTIES OF 1 AND −1

- $1 \times 1 = 1$
- $(-1) \times (-1) = 1$
- $(-1) \times (1) = -1$
- For all real numbers a, $a \times 1 = a$.
- For all real numbers a, $a \times (-1) = -a$.
- For all real numbers a and b, $-1(a \times b) = (-a) \times b = a \times (-b)$. For all real
- numbers a, $-1 \times (-a) = a$.
- For all real numbers a, $a \div 1 = a$.
- For all real numbers a, $a \div (-1) = -a$.
- For any integer n, $n + 1$ is the next larger integer.
- For any integer n, $n + (-1)$ is the next smaller integer.
- The smallest counting number is 1.
- $1 + (-1) = 0$.
- For any real number a, $(1 \times a) + (-1 \times a) = a + (-a) = 0$.

NUMBER PROPERTIES
ODD AND EVEN NUMBERS

$even\ number\ =\ 2n\ where\ n\ is\ an\ integer\ =\ \ldots\ ,\pm 2\ ,0\ ,\pm 2\ ,\ldots$

$odd\ number\ =\ 2k\ +\ 1\ where\ k\ is\ an\ integer\ = \cdots\ ,\pm 1\ ,\pm 1\ ,\ldots$

$6\ \div 0 = {}^{6}/_{0} = Undefined$

even + even = even
even − even = even
even × even = even
odd + odd = even
odd − odd = even
odd × odd = odd
odd + even = odd
even + odd = odd
odd − even = odd
even − odd = odd
odd × even = even
even × odd = even

NUMBER PROPERTIES
PRIMES, MULTIPLES, AND DIVISORS

- $Least\ Common\ Multiple\ = LCM\ (6, 10)\ = 30$ $LCM\ (50, 60)\ = 300$
- $Greatest\ Common\ Divisor\ = GCD\ (6, 10)\ = 2$ (50/60=5/6)$\Longrightarrow GCD\ (50, 60)\ = 10$
- $a \times c\ = b, a$ is a $\boldsymbol{factor}$ $of\ b.$
- $b \times c\ = a, number\ a\ is\ a\ \boldsymbol{multiple}\ of\ a\ counting\ number\ b\ if\ there\ is\ a\ counting\ number\ c$
- $LCM\ (a, b) \leq a \times b$
- $\boldsymbol{divisors}\ of\ 12\ = 1, 2, 3, 4, 6, and\ 12$

  12 is an **improper divisor** of 12

  1, 2, 3, 4, and 6 are the **proper divisors** of 12
- $\boldsymbol{prime\ number}\ = \mathbf{2}, \mathbf{3}, \mathbf{5}, \mathbf{7}, \ldots$
- **composite number** is a counting number greater than 1 such that it has at least three divisors.
- $\boldsymbol{prime\ factorization}\ 24\ = 2 \times 12\ = 2 \times 2 \times 6\ = 2 \times 2 \times 2 \times 3\ = 2^3 \times 3$

SYMBOLS

$=$	equals
$>$	is greater than
$<$	is less than
$\geq$	is greater than or equal to
$\leq$	is less than or equal to
$\neq$	is not equal to
x^2	x squared
x^3	x cubed
$\sqrt{x}$	square root of x
$\lvert x\rvert$	absolute value of x
π	pi (about 3.14)
$+$	addition, plus
$-$	subtraction, minus
$a\times b$	a times b, multiplication
$a.b$	a times b, multiplication
$a * b$	a times b, multiplication
ab	a times b, multiplication
$a \div b$	a divided b, division
a/b	a divided b, division
$a : b$	the ratio of a to b
$\%$	percent

ORDER OF OPERATIONS

You use the order PEMDAS, which stands for **P**arentheses,

Exponents, **M**ultiplication and **D**ivision, and **A**ddition and **S**ubtraction. (***P****lease* ***E****xcuse* ***M****e,* ***D****ear* ***A****unt* ***S****ally*)

- reciprocal of a = 1/a

PROPERTIES OF OPERATIONS

- Commutative Properties ($a + b = b + a$)
- Associative Properties ($a + (b + c) = (a + b) + c$)
- Distributive Property ($a * (b + c) = ab + ac$)
- Identity Properties ($a + 0 = 0 + a = a$, and $a * 1 = 1 * a = a$)
- Inverse Properties ($a + (-a) = (-a) + a = 0$)

FRACTIONS

- A **fraction** is made up of three parts: the **numerator,** the **fraction bar,** and the **denominator.**
- a/b is a **proper fraction** when *a<b.* If *a=b* or *a>b*, you say that it is an **improper fraction.** (5/4 and 1/3)
- **Equivalent fractions** are fractions that are different representations of the same number. (2/6=4/12)

OPERATIONS WITH FRACTIONS

- Adding Fractions
- Subtracting Fractions
- Multiplying Fractions
- Dividing Fractions

DIGITS & DECIMALS

- the integer part (**unit (ones)**, **ten**, **hundred**, **thousand**, **etc**.)
- the decimal part (**tenth, hundredth, thousandth, etc**.) : A decimal is made up of a **whole number** part (which can be a 0), a **decimal point,** and a **decimal fraction.** For example, 2.1345

Billions
Hundred Millions
Ten Millions
Millions
Hundred Thousands
Ten Thousands
Thousands
Hundreds
Tens
Ones
Tenths
Hundredths
Thousandths
Ten Thousandths
Hunderd Thousandths
Millionths

2,293,375,916.975341

Decimal Point

WORD PROBLEMS

Word problems often use keywords that indicate the operation:

Operation	**Keywords**
Addition	Sum, total, all together, and, added to, combined, exceeds, increased by, more than, greater than, plus
Subtraction	Minus, difference, decreased by, less, diminished by, reduced by, subtracted from, deducted
Multiplication	Times, twice, doubled, tripled, product, multiplied by
Division	Quotient, divided by, divide

Ex: In 36 times at bat, Jones got 8 singles, 3 doubles, 1 triple, and 3 home runs. What part of the time did Jones get a hit?

$8 + 3 + 1 + 3 = 15$ hits $\Longrightarrow 15/36 = 5/12$

RATIO & PROPORTION

- A **ratio** is a quotient of two quantities
- A **proportion** is a statement that two ratios are equal. A proportion can be stated as "*a* is to *b* as *c* is to *d*" and written as *a* : *b* : : *c* : *d* or a/b=c/d
- **Properties of Proportions**

$$\frac{a}{b} = \frac{c}{d}$$

$$1.\ ad = bc$$

$$2.\ \frac{b}{a} = \frac{d}{c}$$

$$3.\ \frac{a}{c} = \frac{b}{d}$$

$$4.\ \frac{a+b}{b} = \frac{c+d}{d}$$

$$5.\ \frac{a-b}{b} = \frac{c-d}{d}$$

$$6.\ \frac{a+b}{a-b} = \frac{c+d}{c-d}$$

CONVERSION

convert one unit of measure to another

Time	U.S Customary English System Units	Metric Units
60 minutes = 1 hour	12 inches = 1 foot	10 millimeters = 1 centimeter
60 seconds = 1 minute	3 feet = 1 yard	100 centimeters = 1 meter
24 hours = 1 day	5,280 feet = 1 mile	1,000 meters = 1 kilometer
7 day = 1 week	1760 yards = 1 mile	1,000 milligrams = 1 kilogram
52 weeks = 1 year	16 ounces = 1 pound	1,000 grams = 1 kilogram
12 month = 1 year		1,000 milliliters = 1 liter

MOTION & WORK PROBLEMS

The motion formula *d* = *rt* states that the distance traveled is equal to the rate of travel multiplied by time traveled

Ex: If a bicycle travels at 8 miles per hour for 3 hours, then the bicycle has traveled *d* = 8 miles/hour times 3 hours = 24 miles.

$$1)\ d_T = d_1 + d_2 + d_3 + \cdots \Longrightarrow r_T t_T = r_1 t_1 + r_2 t_2 + r_3 t_3 + \cdots$$

$$2)\ r_T = r_1 + r_2 + r_3 + \cdots \Longrightarrow \frac{d_T}{t_T} = \frac{d_1}{t_1} + \frac{d_2}{t_2} + \frac{d_3}{t_3} + \cdots \text{ if } d_T = d_1 = d_2 = \cdots \Longrightarrow \frac{1}{t_T} = \frac{1}{t_1} + \frac{1}{t_2} + \frac{1}{t_3} + \cdots$$

$$3)\ t_T = t_1 + t_2 + t_3 + \cdots \Longrightarrow \frac{d_T}{r_T} = \frac{d_1}{r_1} + \frac{d_2}{r_2} + \frac{d_3}{r_3} + \cdots \text{ if } d_T = d_1 = d_2 = \cdots \Longrightarrow \frac{1}{r_T} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \cdots$$

MOTION & WORK PROBLEMS

Examples:

A. Sue can paint a room in 4 hours, and Sam can paint the same room in 5 hours. How long would it take them to paint the room together?

$$\frac{1}{t_T} = \frac{1}{t_{Sue}} + \frac{1}{t_{Sam}} \Longrightarrow \frac{1}{t_T} = \frac{1}{4} + \frac{1}{5} \Longrightarrow t_T = \frac{20}{9}$$

B. Abe and Ben, working together, can build a cabinet in 6 days. Abe works twice as fast as Ben. How long would it take each of them to build an identical cabinet on his own?

$$\frac{1}{t_T} = \frac{1}{t_{Abe}} + \frac{1}{t_{Ben}} \;\&\; r_{Abe} = 2r_{Ben} = \frac{1}{t_{Abe}} = \frac{2}{t_{Ben}} \Longrightarrow \frac{1}{6} = \frac{1}{t_{Abe}} + \frac{1}{2t_{Abe}} \Longrightarrow t_{Abe} = 9$$

C. A holding tank can be filled by three pipes (A, B, and C) in 1 hour, 1.5 hours, and 3 hours, respectively. In how many minutes can the tank be filled by the three pipes together?

$$\frac{1}{t_T} = \frac{1}{t_A} + \frac{1}{t_B} + \frac{1}{t_C} = \frac{1}{60} + \frac{1}{90} + \frac{1}{120} \Longrightarrow t_T = 30 \text{ minutes}$$

D. A copy machine makes 40 copies per minute. A second copy machine can make 30 copies per minute. If the two machines work together, how long would it take them to produce 1,400 copies?

$$d_T = d_1 + d_2 = r_1t_1 + r_2t_2 \Longrightarrow 1400 = 40t_{1 or\ 2} + 30t_{1 or\ 2} \Longrightarrow t = 20$$

MOTION & WORK PROBLEMS

Examples:

1. A train left Los Angeles at 8 a.m. headed toward San Francisco. At the same time, another train left San Francisco headed toward Los Angeles. The first train travels at 60 mph, and the second train travels at 50 mph. If the distance between Los Angeles and San Francisco is 440 miles, how long would it take the trains to meet?

$$r_T t_T = r_1 t_1 + r_2 t_2 = 600t + 70t = 440 \Longrightarrow t = 4\ hours$$

2. Two planes left Chicago at the same time, one headed for New York and the other headed in the opposite direction for Denver. The plane headed for New York traveled at 600 mph, while the plane headed for Denver traveled at 150 mph. How long did each plane fly before they were 900 miles apart?

$$r_T t_T = r_1 t_1 + r_2 t_2 = 600t + 150t = 900 \Longrightarrow t = 1.2\ hours$$

3. Two cars left Sioux Falls headed toward Fargo. The first car left at 7 a.m., traveling at 60 mph. The second car left at 8 a.m., traveling at 70 mph. How long would it take the second car to overtake the first car?

$$1)\ d_1 = d_2 \Longrightarrow rt_1 = rt_2 \Longrightarrow 60t_1 = 70t_2 \Longrightarrow t_1 = t_2 - 1 \Longrightarrow t_2 = 7\ hours \Longrightarrow t_1 = 6\ hours$$

4. Carlos can run a mile in 6 minutes, and Kevin can run a mile in 8 minutes. If Carlos gives Kevin a 1-minute head start, how far will Kevin run before Carlos passes him?

$$r_C = \frac{d_C}{t_C} = \frac{1}{6} \Longrightarrow r_K = \frac{d_K}{t_K} = \frac{1}{8} \Longrightarrow d_C = d_K \Longrightarrow r_C t_C = r_K t_K \Longrightarrow \frac{t_C}{6} = \frac{t_K}{8} \Longrightarrow t_{\acute{K}} = t_{\acute{C}} + 1 \Longrightarrow \frac{t_{\acute{C}}}{6} = \frac{t_{\acute{C}} + 1}{8} \Longrightarrow t_{\acute{C}} = 6$$

MOTION & WORK PROBLEMS

Examples:

5. Bev can dig a ditch in 4 hours, and Jan can dig a ditch in 3 hours. If Bev and Jan dig a ditch together, how long would it take?

$$\frac{d_T}{t_T} = \frac{d_1}{t_1} + \frac{d_2}{t_2} \Rightarrow 1 = \frac{1}{t_1} + \frac{1}{t_2} \Rightarrow 1 = \frac{1}{{}^4/_t} + \frac{1}{{}^3/_t} \Rightarrow 1 = \frac{t}{4} + \frac{t}{3} \Rightarrow t = \frac{12}{7}\ hours$$

6. David can type a paper in 5 hours, but when he works with Jim it only takes 2 hours. How long would it take Jim to type the paper alone?

$$\frac{2}{5} + \frac{2}{t} = 1 \Rightarrow t = \frac{10}{3}\ hours$$

7. Kim can paint a barn in 5 days, and Alyssa can paint it in 8 days. Kim and Alyssa start painting a barn together. After 2 days Alyssa gets sick and Kim finishes the work alone. How long did it take Kim to finish painting the barn?

$$\frac{2}{8} + \frac{2+x}{5} = 1 \Rightarrow x = 1.75\ days$$

8. Jay can prune an orchard in 50 hours, and Ray can prune the orchard in 40 hours. How long will it take them to prune the orchard if they worked together?

$$\frac{t}{50} + \frac{t}{40} = 1 \Rightarrow t = \frac{200}{9}\ hours$$

PERCENTAGE & PERCENTILE

A **percent** means parts per hundred.

Price + Add-on = Total Cost.

Original Price – Discount = Selling Price

The key **difference between percentage and percentile is** the **percentage is** a mathematical value presented out of 100 and **percentile is** the per cent of values below a specific value. The **percentage is** a means of comparing quantities.
A **percentile is** used to display position or rank

DIFFERENCE BETWEEN SIMPLE INTEREST AND COMPOUND INTEREST

$Simple\ Interest = P \times r \times t$	$Compound\ Interest = P \times (1 + r) \times t - p$
$P = principal\ amount$ $r = Annual\ interest\ rate$ $t = term\ of\ loan, in\ years$	$P = principal\ amount$ $r = Annual\ interest\ rate$ $t = Number\ of\ years$

ALGEBRA EXPRESSIONS

Algebraic Expression $x^3 - 5xy + 2\mathrm{x}$, $2\mathrm{a}b^2c^5$ & $\frac{5xy+2ab}{5x^2-2a^3}$

Variable x , y , z

Term $2xy$, ${}^{5x^2}/_{3y^3}$

Monomial $3x, 7, x^2 = y$

Binomial $5x + 3y^{-4}$

Trinomial $6 - x^7 + y^{1/2}$

Polynomial $6 - x^7 + y^{1/2} + 4\sqrt{x}$

Coefficient $3y^{-4}$

Like terms $-5x^4y^3$ & $9x^4y^3$

Unlike terms $-5x^2y^3$ & $9x^4y^3$

TABLES OF POWERS & ROOTS

***n*th root**
principal *n*th root

Number	x	16
Square	x^2	$16 \times 16 = 256$
Cube	x^3	$16 \times 16 \times 16 = 4096$
Square Root	$\sqrt{x}$	4
Cube Root	$\sqrt[3]{x}$	≈ 2.52
4th Root	$\sqrt[4]{x}$	2

In $\sqrt[n]{a}$ *,* *n* is the **index**, *a* is the **radicand**, and √ is the **radical sign.**

$$\sqrt[n]{x^a} = (x)^{a/n}$$

$$\sqrt{2} = 1.3$$

$$\sqrt{a^3 b} = a\sqrt{ab}$$

$$\sqrt{3} = 1.7$$

POWERS & ROOTS

5*x*2 has three parts: the 5 is the **coefficient** of *x*2, the *x* is the **base**, and the 2 is the **exponent**.

Laws of Exponents

$$1.\, x^a x^b = x^{a+b}$$

$$2.\, (x^a)^b = x^{ab}$$

$$3.\, \frac{x^a}{x^b} = x^{a-b} \qquad x \neq 0$$

$$4.\, (xy)^a = x^a \times y^a$$

$$5.\, \left(\frac{x}{y}\right)^a = \frac{x^a}{y^a} \qquad y \neq 0$$

$$6.\, x^{-a} = \frac{1}{x^a} \qquad a > 0, \qquad x \neq 0$$

$$7.\, x^0 = 1 \qquad x \neq 0$$

POWERS & ROOTS

Scientific notation is a way to write very large or very small numbers by rewriting them as numbers between 1 and 10 multiplied by a power of 10.
$265{,}000$ is written as 2.65×10^{5}, and 0.000148 is written as 1.48×10^{4}.

We use exponents to write numbers in **expanded notation**.

$$265 \;= 200 + 60 + 5 = 2\times10^{2} \;+\; 6\times10^{1} + 5\times10^{0}$$
$$0.45 = 0.4 + 0.05 = 4\times10^{-1} + 5\times10^{-2}$$
$$24.3 = 20 + 4 + 0.3 = 2\times10^{1} \;\; + 4\times10^{2} \;\; + 3\times10^{-1}.$$

- *In* $\sqrt[n]{a}$, *n* is the **index**, *a* is the **radicand**, and √ is the **radical sign.**

FUNCTIONS

relation

domain

range

function

Sometimes a special symbol such as ◊, □, #, or * may be used to denote the rule for a function.

$$f(x) = x\ \lozenge = x\ \square = x\ \# = x\ *$$

$$f(x, y) = x \lozenge y = x \square y = x \# y = x * y$$

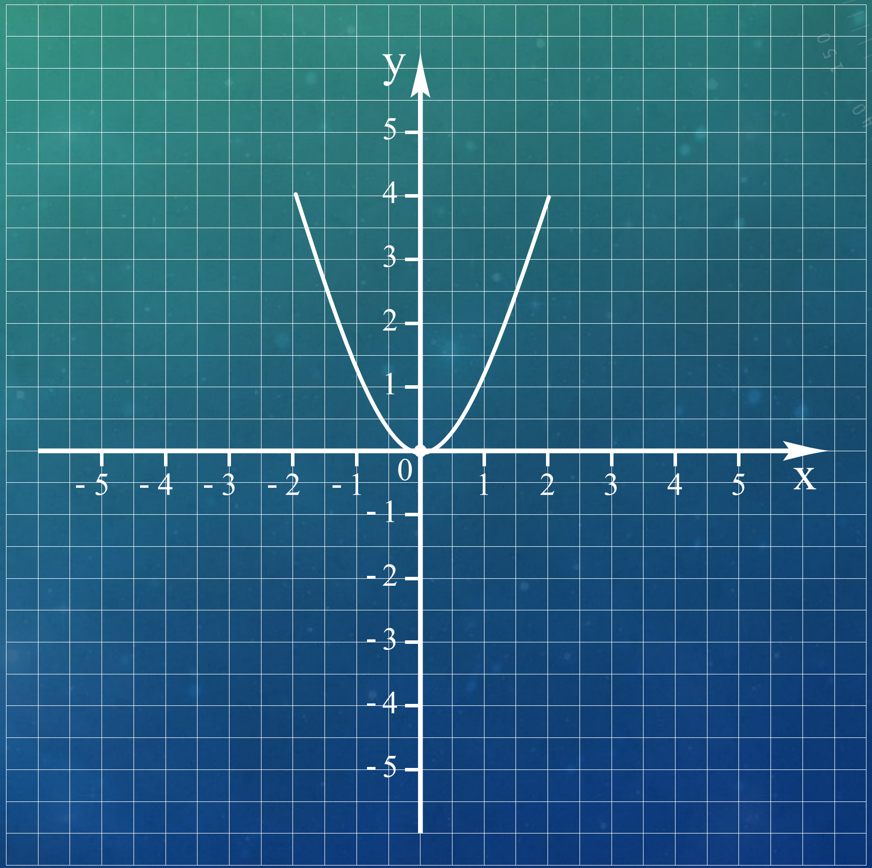
QUADRATIC EQUATIONS & INEQUALITIES & PARABULA

quadratic equation in one variable $\boldsymbol{ax^2 + bx + c = 0 \quad if\ a \neq 0}$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $b^2 \leq\ 4ac$ is negative, the quadratic equation has no real roots.

$$\frac{-b}{a} = x_1 + x_2 \qquad \frac{c}{a} = x_1 x_2$$



TYPES OF AVERAGE

- **AVE** = SUM ÷ N
- **mode** of a set of values is the value that occurs most often (2, 8, 7, 6, 4, 3, 8, 5, 1 mode=8)
- **median** is the middle value, or the average of the two middle values (24, 6, 7, 23, 13, 12, 18 median=13). The median of 1, 3, 4, 7, 10, 20 is the average of the two middle values 4 and 7: (4 + 7) / 2 = 5.5
- **weighted averages:** the average of two or more averages

  SUM = $hA + iB + jC$ and $N = h + i + j$. AVE = $(hA + iB + jC) \div (h + i + j)$.
- **range =** R = MAX − MIN

MEAN VS. AVERAGE

- What is the mean versus average? Most people understand the word **average** as describing a representative value within a group.
- For example, the average age of a group of three people aged 10, 16 and 40 is (10 + 16 + 40) / 3, or 22.
- When speaking statistically, this average age of 22 is referred to as the **mean age**.

TYPES OF AVERAGE

Example of weighted averages:

The average traffic fine was \$90 on Monday for the 18 people ticketed, the average fine was \$76 on Tuesday for the 20 people ticketed, and the average fine was \$80 on Wednesday for the 24 people ticketed. What was the average fine for the 3 days?

SUM = 18(\$90) + 20(\$76) + 24(\$80) = \$1,620 + \$1,520 + \$1,920 = \$5,060

N = 18 + 20 + 24 = 62

AVE = \$5,060 ÷ 62 = \$81.6129 = \$81.61

AVERAGE SPEED

- $x = vt + x_0 \Longrightarrow v =$ average speed
- $x = at^2 + vt + x_0 \Longrightarrow v_t =$ average speed $in\ t = 2at + v$

NOT GROUPED YET

- time of 21 runners
- 59, 65, 61, 62, 53, 55, 60 , 70, 64, 56, 58, 58, 62, 62, 68, 65, 56, 59, 68, 61, 67
- $Average = \frac{59+65+61+62+53+55+60+70+64+56+58+58+62+62+68+65+56+59+68+61+67}{21} = 61.38$
- 53, 55, 56, 56, 58, 58, 59, 59, 60, 61, 61 , 62, 62, 62, 64, 65, 65, 67, 68, 68, 70
- **Median = 61**
- **Mode = 62**

GROUP FREQUENCY TABLE (INTERWAL AND FIX)

second	Frequency
51-55	2
56-60	7
61-65	8
66-70	4

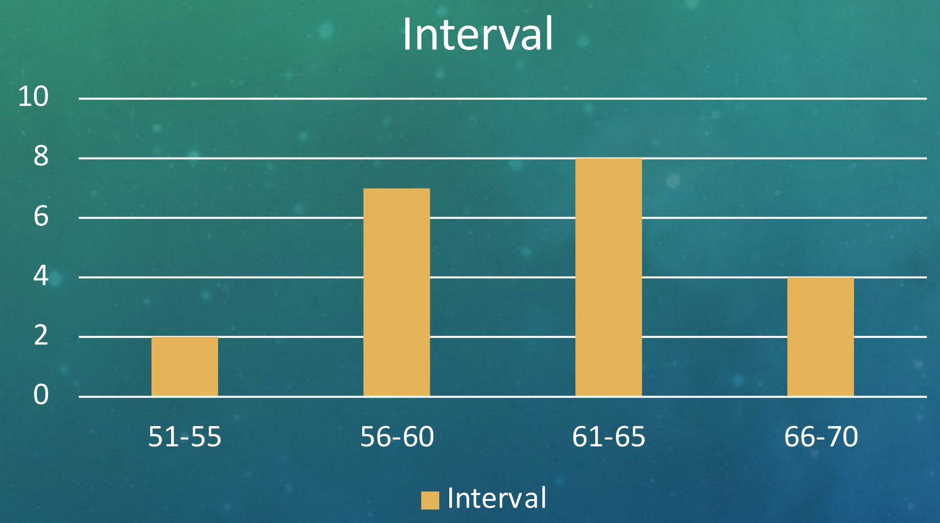
second	Midpoint	Frequency
51-55	53	2
56-60	58	7
61-65	63	8
66-70	68	4

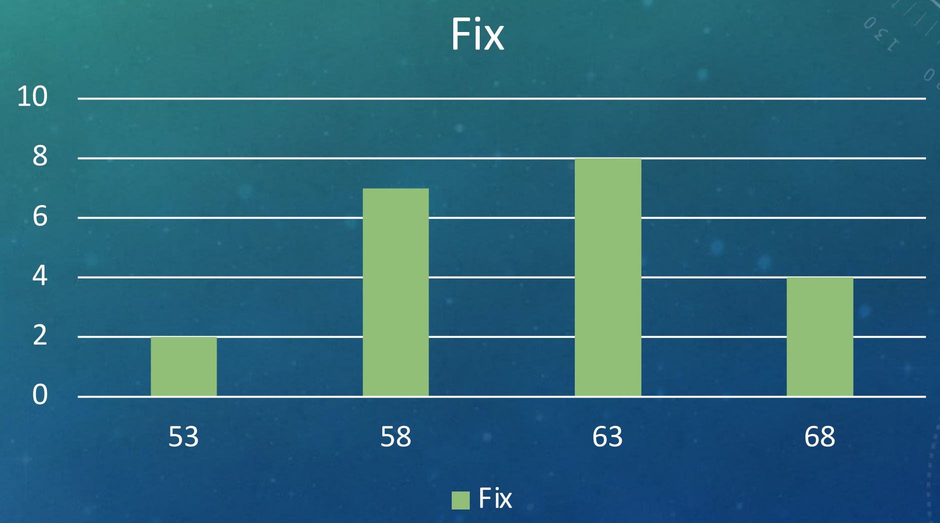
$$Approximate\ Average = \frac{53\times2 + 58\times7 + 63\times8 + 68\times4}{2+7+8+4} = \frac{1288}{21} = 61.33$$

$$Mode = 8$$

$Median\ range$: The median is the number in the middle, here is the 11th number, in the 61−65 group:
We can say: "The **median group** is 61−65"

MEDIAN IN HISTOGRAM (INTERVAL AND FIX)





MEDIAN IN CHARTS AN HISTOGRAM

- https://www.khanacademy.org/math/ap-statistics/summarizing-quantitative-data-ap/measuring-center-quantitative/v/median-histogram
- https://www.youtube.com/watch?v=aFpHDurNGoU
- https://www.youtube.com/watch?v=reiu1NZhRiE
- https://www.youtube.com/watch?v=xTwDmnEEb9E

INTERQUARTILE RANGE & BOX PLOTS

Sometimes we want to look at the way the data spread out with greater detail than the range provides.

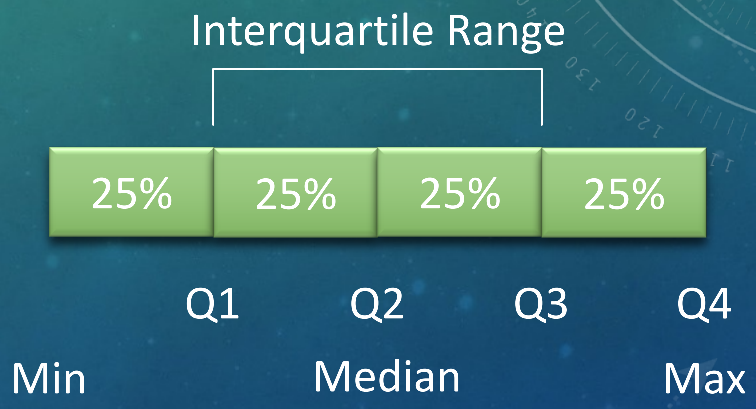
Example:

A. 9, 11, 10, 5, 13, 21, 23

The ordered values are 5, 9, 10, 11, 13, 21, 23.
The middle value of the seven values is 11. The median (MD) is 11.
The values from the minimum (MIN) to the median (MD) are 5, 9, 10, 11. The two middle values are 9 and 10.

(9 + 10) ÷ 2 = 9.5 = Q1.
The values from the median to the maximum (MAX) are 11, 13, 21, 23. The two middle vales are 13 and 21.

(13 + 21) ÷ 2 = 17 = Q3.
The interquartile range is IR = Q3 – Q1 = 17 – 9.5 = 7.5



INTERQUARTILE RANGE & BOX PLOTS

Example:

B. 105, 114, 134, 127, 219, 97

The ordered values are 97, 105, 114, 127, 134, 219.
The middle values of the six values are 114 and 127. (114 + 127) ÷ 2 = 120.5. The median (MD) is 120.5.
The values from the minimum (MIN) to the median are 97, 105, 114.
The middle value is 105. Q1 = 105.
The values from the median to the maximum (MAX) are 123, 134, 219. The middle value is 134. Q3 = 134.
The interquartile range is IR = Q3 – Q1 = 134 – 105 = 29.

INTERQUARTILE RANGE & BOX PLOTS

Example:

C. 5, 7, 8, 3, 19, 14, 16, 17, 11, 9, 4, 3, 6, 8

The ordered values are 3, 3, 4, 5, 6, 7, 8, 8, 9, 11, 14, 16, 17, 19.
The middle two values of the fourteen values are 8 and 8. (8 + 8) ÷ 2 = 8 and the MD = 8.
The values from the MIN to the MD are 3, 3, 4, 5, 6, 7, 8.
The middle value is 5. Q1 = 5.
The values from the MD to the MAX are 8, 9, 11, 14, 16, 17, 19.
The middle value is 14. Q3 = 14. The interquartile range is IR=Q3 –Q1 =14–5=9.

INTERQUARTILE RANGE & BOX PLOTS

Example:

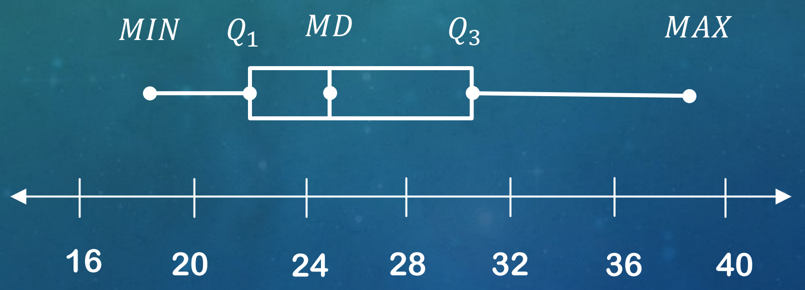
D. 25, 2, 5, 6, 7, 6, 23, 18, 7, 10, 15, 21, 28

The ordered values are 2, 5, 6, 6 7, 7, 10, 15, 18, 21, 23, 25, 28. The middle value of the thirteen values is 10. The MD = 10. The values from the MIN to the MD are 2, 5, 6, 6, 7, 7, 10. The middle value is 6. Q1 = 6. The values from the MD to the MAX are 10, 15, 18, 21, 23, 25, 28. The middle value is 21. Q3 = 21.
The interquartile range is IR=Q3 –Q1=21–6=15.

INTERQUARTILE RANGE & BOX PLOTS

Example:

Make a box-and-whisker plot for a set of values that has 18, 22, 25, 30, and 39 as its five-number summary.



INTERQUARTILE RANGE & BOX PLOTS

Example:

If a set of data consists of only the first ten positive multiples of 5, what is the interquartile range of the set?

A)15
B) 25
C) 27.5
D) 45

S = [5, 10, 15, 20, 25, 30, 35, 40, 45, 50]

Total elements are 10 so the Median is the average of 5th & 6th term which is 27. 5.

This splits our set into 2 halves one being lower and upper the median each having 5 terms, again the median for this set is the 3rd term ((5 + 1)/ 2 = 3)

Hence 40 - 15 = 25

Also keep in mind whenever you have even number of terms the IQRs will be simply the difference of the median of two halves

INTERQUARTILE RANGE & BOX PLOTS

The below box plot is for the following set **S** of points:
S = {1, 2, 3, 4, 4, 8, 8, 8, 8, 9, 9, 10, 10, 10, 10, 10}

Question: What is the interquartile range for **S**?

A) 1

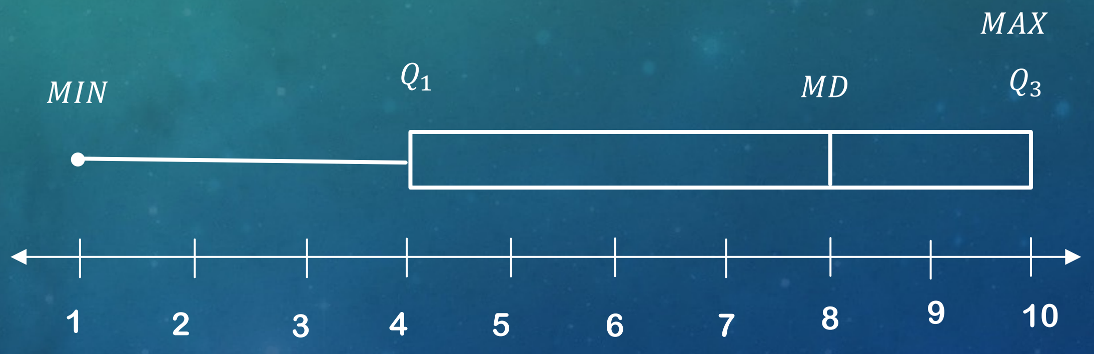
B) 4

C) 5

D) 6

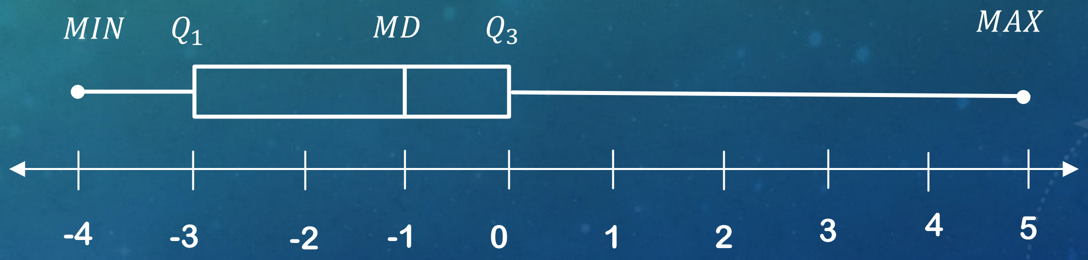
E) 9

Answer: D



INTERQUARTILE RANGE & BOX PLOTS

Question: Which of the following sets of data applies to this box-and-whisker plot?



A) -4, -4, -2, 0, 0, 5

B) -4, 1, 1, 3, 4, 4

C) -4, -4, -3, 1, 5

D) -5, 3, 4, 5

E) -4, -4, -2, -2, 0, 0, 0, 5

Answer: E

FREQUENCY DISTRIBUTIONS

$$NUM \;=\; sum\ f\ and\ sum\ (X - AVE)^2 = \; sum\ [f(X - AVE)^2]$$

Example:

Find the mean, median, and mode for the set of values presented in this frequency distribution.

SUM = 8(5) + 2(6) + 9(9) + 6(11) + 3(12) + 1(14) + 4(15)= 309

NUM = 8 + 2 + 9 + 6 + 3 + 1 + 4 = 33

AVE = SUM ÷ NUM = 309 ÷ 33 = 9.363636 . . . ≈ 9.36

Value	Frequency
5	8
6	2
9	9
11	6
12	3
14	1
15	4

FREQUENCY DISTRIBUTIONS

$$NUM = sum\ f\ and\ sum\ (X - AVE)^2 = sum\ [f(X - AVE)^2]$$

Example:

Find the mean, median, and mode for the set of values presented in this frequency distribution.

SUM = 6(4.5) + 4(8.5) + 9(12.5) + 8(16.5) = 305.5

NUM = 6 + 4 + 9 + 8 = 27

AVE = SUM ÷ NUM = 305.5 ÷ 27 ≈ 11.31

Interval	Frequency	Midpoint of interval
3-6	6	4.5
7-10	4	8.5
11-14	9	12.5
15-18	8	16.5

STANDARD DEVIATION

$$SD = \sqrt{\frac{Sum\ Sq}{NUM}} = \sqrt{\frac{Sum\ (X - AVE)^2}{NUM}}$$

Example:

Find the standard deviation for each set of values.

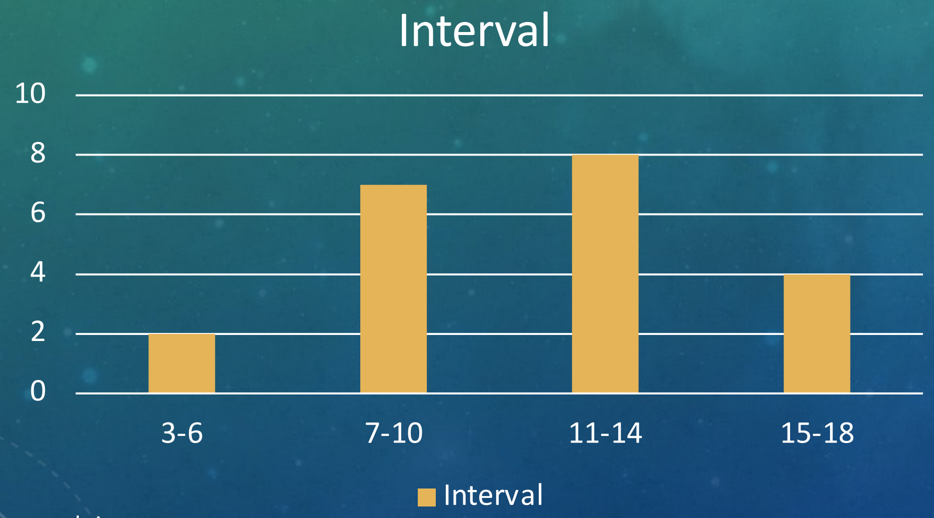
B. 1, 6, 13, 20

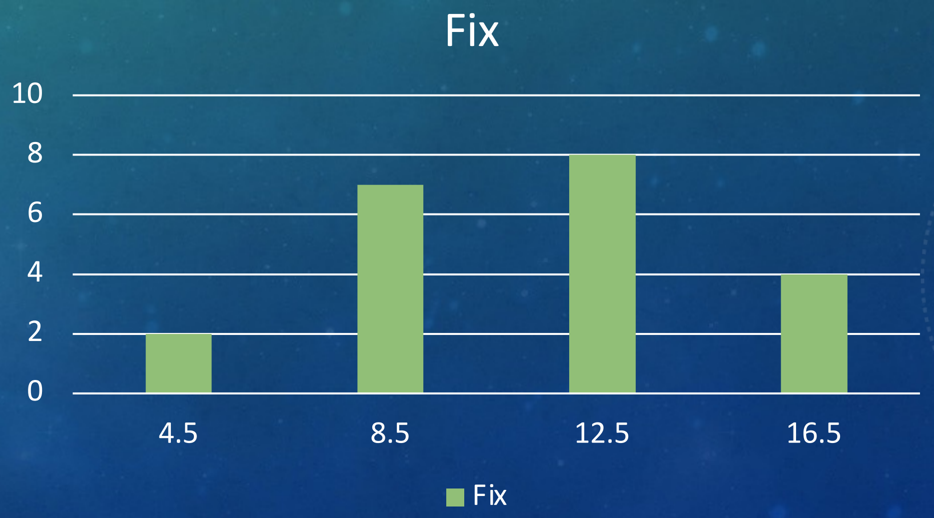
$$AVE = \frac{1 + 6 + 13 + 20}{4} = \frac{40}{4} = 10$$

$$SD = \sqrt{\frac{(1-10)^2+(6-10)^2+(13-10)^2+(20-10)^2}{4}} = \sqrt{\frac{206}{4}} \approx 7$$

STANDARD DEVIATION (INTERVAL TABLE AND HISTOGRAM)

Interval	Frequency	Midpoint of interval
3-6	6	4.5
7-10	4	8.5
11-14	9	12.5
15-18	8	16.5





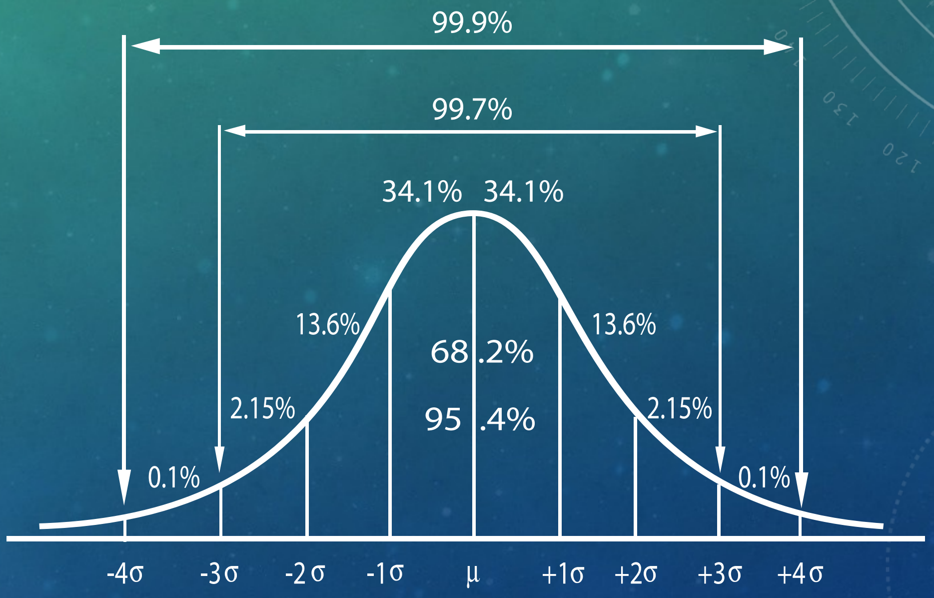
TYPES OF AVERAGE & STANDARD DEVIATION

	First	$\mp k$	$\times k$
Mode	Mode	$Mode \mp k$	$Mode \times k$
Average (Mean)	A	$A \mp k$	$A \times k$
Min	Min	$Min \mp k$	$Min \times k$
Max	Max	$Max \mp k$	$Max \times k$
Median	M	$M \mp k$	$M \times k$
Range	R	$R \mp k$	$R \times k$
Interquartile Range	IR	$IR \mp k$	$IR \times k$
Standard Deviation	SD	SD	$SD \times k$

NORMAL DISTRIBUTION OR BELL CURVE

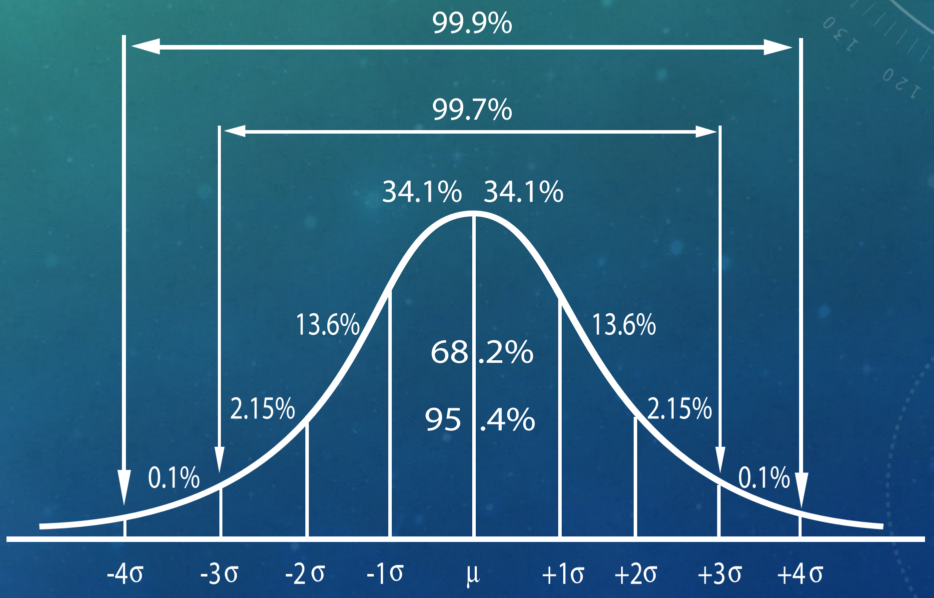
Where m is the mean, and m+d and m-d designate one standard deviation greater and less than the mean. The percentage values in the above figure indicate what percentage of the data fall into that region. GRE data analysis questions that measure your understanding of the normal distribution may or may not provide you with a plot (as above).

•The mean, median, and mode of the data are nearly all equal



NORMAL DISTRIBUTION OR BELL CURVE

- The area under a normal curve is 100%.
- 50% of the values are above the mean and 50% are below.
- About 68% of the values are within 1 standard deviation of the mean.
- About 95% of the values are within 2 standard deviations of the mean.
- About 99.7% of the values are within 3 standard deviations of the mean.
- The mean, median, and mode of the data are nearly all equal



NORMAL DISTRIBUTION OR BELL CURVE

Example

The average men's height in the US is 70 inches ± 3 inches.

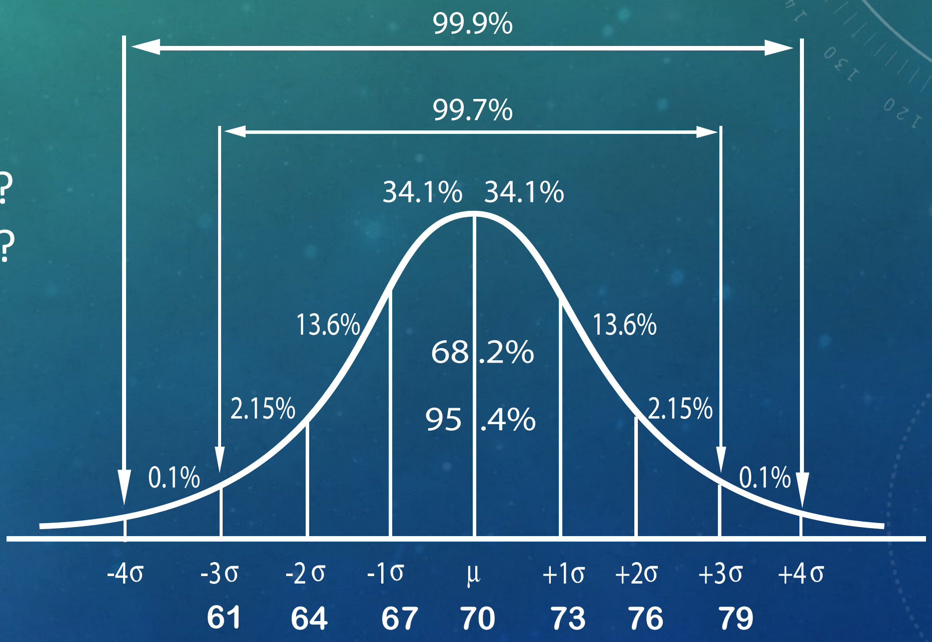
(a) What percent of men are between 67 and 73 inches tall?
(b) What percent of men are between 64 and 70 inches tall?
(c) What percent of men are taller than 79 inches?

Answers:

(a) 68%

(b) 47.5%

(c) 0.15%.



NORMAL DISTRIBUTION OR BELL CURVE

The random variable X is normally distributed. The values 650 and 850 are the 60th and 90th percentiles of the distribution of X, respectively.

Quantity A

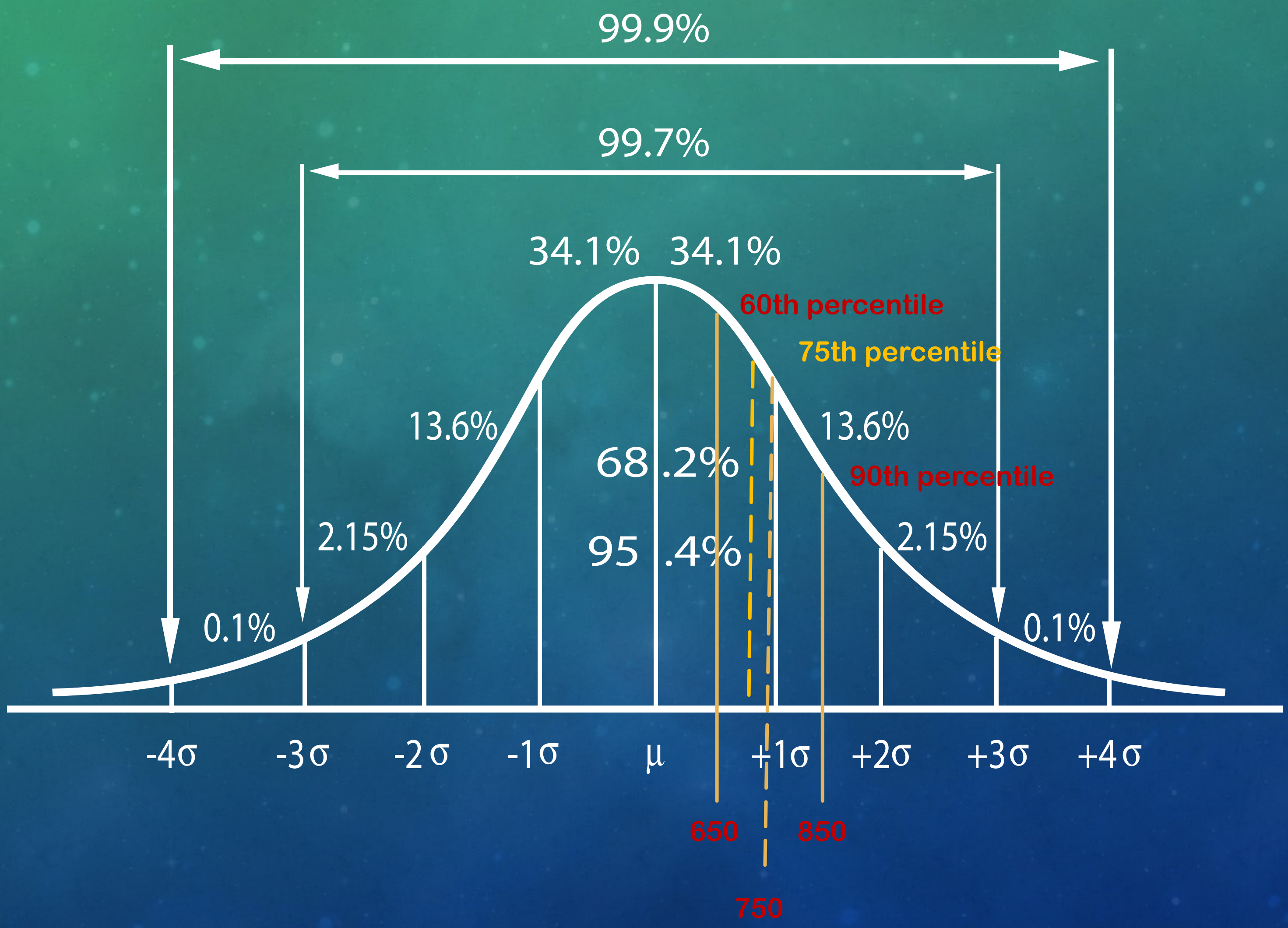
The value at the 75th percentile of the distribution of X

Quantity B

750

A. Quantity A is greater.
B. Quantity B is greater.
C. The two quantities are equal
D. The relationship cannot be determined from the information given.

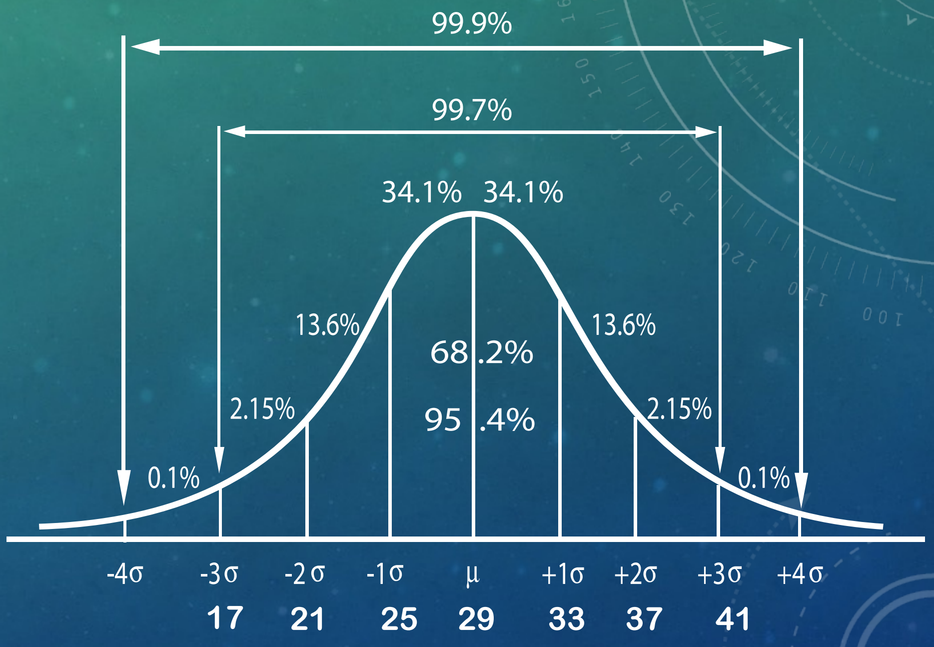
NORMAL DISTRIBUTION OR BELL CURVE



Answer: B

NORMAL DISTRIBUTION OR BELL CURVE

- **Question:** Stats-R-Us laboratories has conducted a survey to determine how many strawberries are eaten by 100,000 people during a one year period. The data indicate that the number of strawberries eaten is approximately normally distributed with a mean of 29 strawberries, and a standard deviation of 4 strawberries eaten by each person. According to this data, approximately how many of the surveyed people ate more than 25 strawberries during the course of the year?
- 16,000
- 48,000
- 50,000
- 68,000
- 84,000



SIMPLE PROBABILITY

Example:

A bag contains 12 red marbles and 18 blue marbles. Two marbles are selected at random from the bag. What is the probability that neither marble will be blue?

There are 12 + 18 = 30 marbles in the bag. There are 12 × 11 = 132 ways to get 2 red (not blue) marbles. There are 30 × 29 = 870 ways to get 2 marbles from the bag.

$$P(not\ blue) = \frac{132}{870} = \frac{22}{145}$$

SIMPLE PROBABILITY

Eight points are equally spaced on a circle. If 3 of the 8 points are to be selected at random, what is the probability that a triangle having the 3 points chosen as vertices will be a right triangle?

A) 1/14
B) 1/7
C) 3/14
D) 3/7
E) 6/7

Answer = 24/56 = 3/7

ARITHMETIC SEQUENCE

An **arithmetic sequence** is a sequence of numbers such that the difference between the consecutive terms is constant.

$$a_1 + a_2 + \cdots + a_n = \frac{n}{2}(a_1 + a_2) = \frac{n}{2}(2a_1 + (n-1)d)$$

$$a_n = a_1 + (n-1)d$$

GEOMETRIC SEQUENCE

$$a + ar + ar^2 + \cdots + ar^{(n-1)} = \sum_{k=0}^{n-1} ar^k = a(\frac{1-r^n}{1-r})$$

NUMBER SEQUENCES

- For any positive integer n, the sum of the first n positive integers equals (n(n + 1)) / 2. What is the sum of all the even integers between 49 and 101?
- A. 1875
- B. 1950
- C. 2550
- D. 4950
- E. 5000

PERMUTATIONS & COMBINATIONS

Daisy wants to arrange four vases in a row outside of her garden. She has eight vases to choose from. How many vase arrangements can she make?

- 1680
- 24
- 70
- 8
- 32

$$P(n,k) = \frac{n!}{(n-k)!} = \frac{8!}{(8-4)!} = 1680$$

PERMUTATIONS

Example:

- **In how many ways can five different colored balls be arranged in a row?**
- 25
- 20
- 15
- 225
- 120
-

  Answer: 5! = 5 * 4 * 3 * 2 * 1 = 120

COMBINATIONS

There are 300 people at a networking meeting. How many different handshakes are possible among this group?

- None of the other answers
- 45,000
- 300!
- 44,850
- 89,700

Answer: $C(n, k) = \frac{n!}{(n-k)! \times k!} = \frac{300!}{(300-2)! \times 2!} = 44{,}850$

THANK YOU

SHAYESTEH.IR

2021